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Left Nucleus In Semiprime Strongly (-1, 1) Rings

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ABSTRACT: In this paper we have to prove that if R is a semi prime strongly (-1,1)ring with characteristic ≠2,3 then left nucleus is equal to the centre $N_{\alpha} = C$

KEY WORDS: Semi prime ring, strongly (-1,1) ring, nucleus, left nucleus, centre

INTRODUCTION: In [1] Kleinfeld prove that if a prime alternative ring R is not associative then the nucleus and centre C are equal. Slater [2] sharpened this by substituting semi prime and purely non associative for prime. The variety of right alternative rings one can have $N_{\alpha} \neq N$ showed by Miheev [3,4,5] had constructed a simple ring and also in a finite dimensional prime algebra , particularly in [4] Miheev had constructed a simple right alternative nil ring R, that is not alternative whose elements C and b_n (k) are in the left annihilator of R and so naturally belong to the left nucleus N_{α}.In general in any simple nil ring the centre $C = 0$ $N = C$ in R . Thus $N_{\alpha} \neq N = 0$ In this simple right alternative ring. In [5] Miheev considerably constructed a 17 dimensional prime right alternative algebra which is not alternative. Among other elements which are in N_{α} but not in N_{β} How ever we now show that $N_{\alpha} = C$ in any semi prime strongly (-1,1) ring with characteristic ≠2,3.

PRELIMINARIES: Let R be a non associative ring. we shall denote the associator and commutator by $(pq) = pq - qp$

$$(pqr) = (pq)r - p(qr) \text{ for all } p,q,r \text{ in } R$$

A ring is called right alternative if it satisfies the identity $(q, p, p) = 0$ which satisfies also the identity $(p,q,q) = 0$ is called alternative and one which satisfies on linearization getting($(p,q,r)+(p,r,q) = 0$ along with the identity $((p,q),r) = 0$ is called strongly (-1,1).

The following are the notations are used for nuclei and centers in right alternative ring R ,

Left nucleus, $N_{\alpha} = \{\alpha \in R/(\alpha,\beta,\beta) = 0\}$

Middle nucleus $N_\beta = \{\alpha \in R / (\beta, \alpha, \beta) = 0\}$

Right nucleus, $N_\gamma = \{\alpha \in R / (\beta, \beta, \alpha) = 0\}$

Associative centre or Nucleus $N = N_\alpha \cap N_\gamma$

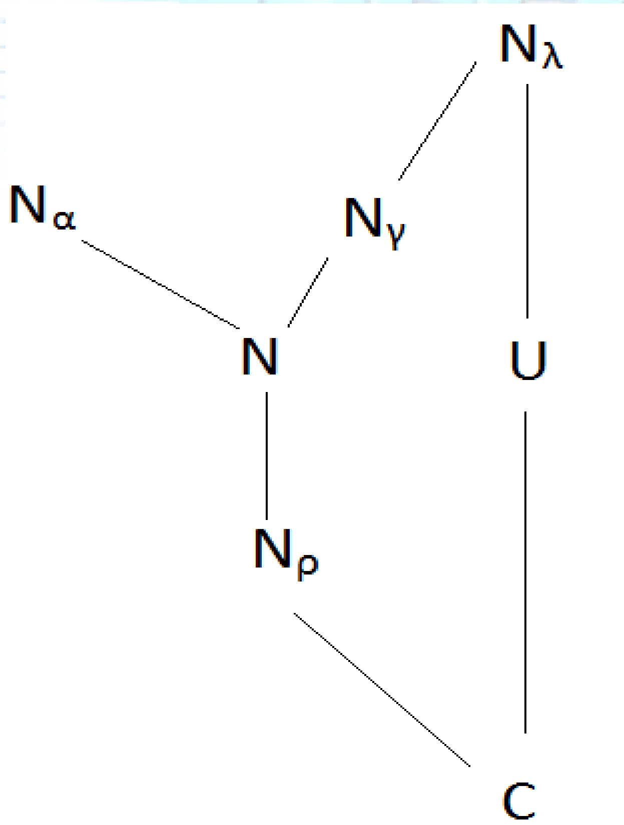
The right alternative nucleus $N_\lambda = \{v \in R / (P, P, V) = 0\}$

Alternative centre $N_\rho = \{v \in R / (p, v, p) = 0, (v, p, q) = (p, q, v) = (q, v, p)\}$

Commutative centre $U = \{u \in R / (u, R) = 0\}$

The associative commutative centre (or) Centre $C = N \cap C$

The following diagram gives a rough idea related to nuclei and centers.



A right alternative ring R is said to be prime if the product of two ideals is zero if ad only if at least one of the ideals is zero i.e. if AB=0 for ideal Aand Bof R implies either A=0 or B=0.Also a ring R is semi prime if the only ideal of R which squares to zero is the zero ideal i.e if $I^2=0$ for I is an ideal of R implies I=0 then R is semi prime.

Now we show that $N_\alpha = C$ in semi prime strongly (-1, 1) ring.

Through this paper R is assumed to be 2, 3- divisible semi prime strongly (-1, 1) ring. The following identity valid in any ring known as Teichmuller identity,

1) $(\omega p, q, r) - (\omega, pq, r) + (\omega, p, qr) = \omega(p, q, r) + (\omega, p, q)r.$

From this identity, we have $N_\alpha, N_\beta, N_\gamma$ which are associative sub rings of R. The proofs are as follows

$$(\omega p, q, r) - (\omega, pq, r) + (\omega, p, qr)$$
$$((\omega p)q)r - (\omega p)(qr) - (\omega(pq))r + \omega((pq)r)) + (\omega p)(qr) - \omega(p(qr))$$

$$((\omega p)q)r - (\omega(pq))r + \omega((pq)r)) - \omega(p(qr))$$

$$((\omega p)q)r - (\omega p)(qr) - ((\omega(pq))r - \omega((pq)r)) + ((\omega p)(qr) - \omega(p(qr)$$
$$((\omega p)q - \omega(pq))r + \omega((pq)r - p(qr)$$

$$= (\omega, p, q)r + \omega(p, q, r)$$

Left nucleus is a sub ring of R as

Let $\alpha_1 \in N_{\alpha,}$ and $\alpha_2 \in N_\alpha$

Then $\alpha_1 - \alpha_2 \in N_\alpha$ now using (1)

$$(\alpha_1\alpha_2, \beta, \beta) - (\alpha_1, \alpha_2\beta, \beta) + (\alpha_1, \alpha_2, \beta\beta) = \alpha_1(\alpha_2, \beta, \beta) + (\alpha_1, \alpha_2, \beta)\beta$$
$$\Rightarrow \alpha_1, \alpha_2 \in N_\alpha$$

Hence left nucleus is a sub ring of R

The following are required interesting points

Assume that $n \in N_\alpha$ and then $\omega = n$, getting

$(np, q, r) = n(p, q, r) \forall p, q, r \in R.$ ----------------------(2)

Similarly, $n \in N_\gamma$,then put $r = n,$ getting

$(\omega, p, qn) = (\omega, p, q)n \forall p, q, r \in R.$ ---------------------(3)

If $n \in N_\alpha \cap N_\beta$, then put $p = n$, getting

$(\omega n, q, r) = (\omega, nq, r) \forall p, q, r \in R.$ ----------------------(4)

If $n \in N_\beta \cap N_\gamma$,then put $q = n,$ getting

$(\omega, pn, r) = (\omega, p, nr) \forall p, q, r \in R$ ----------------------(5)

Since the commutator belongs to the ring,eq (2) can be written as

$(np - pn, q, r) = n(p, q, r)$

$(np,q,r)-(pn,q,r)=n(p,q,r),$

$(np,q,r)=(pn,q,r)=n(p,q,r).$ --------------------------(6)

Along with the above the following identities are useful in further proofs, they are

7) $(q,p,r)+(q,r,p)=0$

8) $(((\omega,p),q),r)=0$

9) $(p,q)^2=3(p,q(p,q))$

10) $(u,p,q)=2(p,q,u)=2(q,u,p)=2(q,p,u)=2(p,u,q)$

11) $(p(q,r),u)=0=(p,(q,r,u))$

12) $((p,q),r)^2=0$

13) $(R,R,U)\subseteq U,(R,U,R)\subseteq U,(U,R,R)\subseteq U$

14) $(U,R,R)((R,R),R)=0$

15) $[(\omega,p),q,r]-[\omega,(p,q,r)]+[p,(\omega,q,r)]=[p,\omega[q,r]]-[\omega,p,[q,r]]$

16) $(N_\alpha\cap U)=(N_\gamma\cap U)=C$

Lemma: If $T=\{t\in N_\alpha / t(R,R,R)=0\}$ then T is an ideal of R and T(R,R,R)=0

Proof: By substituting t for n in (11)

$(tp,q,r)=t(p,q,r)=(pt,q,r)=0$

Thus $tr\subset N_\alpha$ and $rt\subset N_\alpha$

Suppose that $t\in T$ and $\omega\in R$

First observe that $t\omega.(p,q,r)=t.\omega(p,q,r)$

From equation (1) multiplied with t on left side yields

$(\omega p,q,r)-(\omega,pq,r)+(\omega,p,qr)=\omega(p,q,r)+(\omega,p,r)r$

$t.\omega(p,q,r)=-t.(\omega,p,q)r=-t(\omega,p,q).r=0$

Thus $t\omega.(p,q,r)=0$

however from(12) and (11) yields

$(q,r)(p,r,s)=-(p,r)(q,r,s)$

$(t,\omega)(p,q,r)=-(p,\omega)(t,q,r)=0$

$=-(-(\omega t)(p,q,r)=0$

$=\omega t.(p,q,r)=0$

Using $t\omega(p,q,r)=0$

We obtain $\omega t.(p,q,r)=0$ thus T is an ideal of R.

Theorem: If R is a semi prime then R satisfies the identity ((p,q),r) =0 which is strongly $(-1,1)$.

Proof: Define $T(J)=\{t\in R/JR=0=tj\}$.

Let K be the ideal generated by $((R,R),R),R,R)$ using (8) implies $((R,R),R)\subset U$ so that $K\subset J$.

From (14) it follows that $(U,R,R)((R,R,),R)=0.$

Then (10) and the characterstic ≠2,3 yield.

$(R,R,U)((R,R),R)=0$

using (13) we have $(p,q,u)\in u$

So that $(p,q,u)r=r(p,q,u)$

But (11) implies $(a,(b,c),u')=0$

Thus $(t,u',((b,c),d)=0$

Let $u'=(p,q,u)$

Thus $r(p,q,u).((b,c),d)=(r,u'((b,c),d)=0.$

This show $(p,qu)r.((b,c),d)=0$

This sufficies to shows $J.((b,c),d)=0$

So that $((R,R),R)\subset T(J)$ is an ideal.

It follows that $((R,R)R),R,R)\subset T(J)$ and so $k\subset T(J)$

but now $T(J).J=0$

$\Rightarrow K^2=0$

Using semi prime it follows that K=0 thus $((R,R),R)$ lies in the left nucleus using (10) the charactestic $\neq$2,3, then $((R,R),R)$ lies in the nucleus and then by (8) also in the centre of R.

From (12) we know that $((p,q)r)^2=0$

If L is the ideal generated by $((p,q),r)$ then $L^2=0$ implies L=0

Using semi prime, this concludes the proof of the theorem.

MAIN RESULT:

If R is semi prime strongly (-1,1) ring with characteristic $\neq$2,3 then $N_\alpha=C$.

PROOF: In a (-1,1) ring with characteristic$\neq$2,3 and from above lemma $[N_\alpha,(R,R,R)]=0$ thus for $n\in N_\alpha$ by (15)

$$([p,n],q,r)=[p,(n,q,r)]-[n,(p,q,r)]+(n,p[q,r])-(p,n,[q,r])$$

Now since R is a strongly (-1,1) ring ,$[q,r]\in u\subseteq N_\lambda$

Hence $[(p,n).q,r]=-(p,n[q,r])=(n,p,[q,r])=0$

And so from (16) $[p,n]\in N_\alpha\cap U=C$

By using the identity (16) in strongly (-1,1) ring with characteristic $\neq$2,3

We have $[p,q]^3=0$

Which leads $[p,q]^2\in C$ generates a trivial ideal and so $[p,n]=0$

Thus it follows $N_\alpha\subseteq(N_\alpha\cap U)=C$

By using (16) $N_{\alpha} = C.$

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